Grade 7 Divisibility Rules

7.N.1	
Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and why a number cannot be divided by 0.	1. Determine if a given number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10 and explain why. 2. Sort a set of numbers based upon their divisibility using organizers, such as Venn and Carroll diagrams. 3. Determine the factors of a number using the divisibility rules. 4. Explain, using an example, why numbers cannot be divided by 0.

Clarification of the outcome:

- The outcome concerns divisibility rules for 2 through 10, except for 7. There is a divisibility rule for 7 but it is not "nice" and not particularly fast to use.
- The outcome also concerns division by zero. This cannot be done (is not defined) for the real number system (numbers like, 3, -5, -11/1, SQR 2). Division by zero is defined in some "strange" systems of higher mathematics. Note that, while division by zero is undefined for real numbers, division into zero (e.g. $0 \div 5$) is defined; the result is zero.

Required close-to-at-hand prior knowledge:

- Proficiency with mental arithmetic, including basic facts.
- Understand division as splitting up into equal groups.
- Understand factors and primes.

Note:

- Divisible by 2: ones digit is 0, 2, 4, 6 or 8 (the number is even).
- Divisible by 3: sum of the digits is divisible by 3.
- Divisible by 4: 2-digit number formed by the ones & tens digits is divisible by 4.
- Divisible by 5: ones digit is 0 or 5.
- Divisible by 6: divisible by 2 AND divisible by 3.
- Divisible by 8: 3-digit number formed by the ones, tens, & hundreds digits is divisible by 8.
- Divisible by 9: sum of the digits is divisible by 9.
- Divisible by 10: ones digit is 0.

SET SCENE stage

The problem task to present to students:

Ask students to research codes such as UPC bar codes and bank account number codes. Ask them to pay attention to what are referred to as CHECK digits. One URL is:

HowStuffWorks "How UPC **Bar Codes** Work"

Comments:

The main purpose of the task is to get students thinking about “real world” uses of divisibility checks. Sometimes a divisibility check is used to confirm accurate entry of a numerical code.

DEVELOP stage

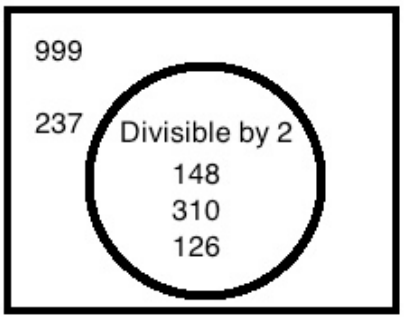
Activity 1: Revisits SET SCENE.

- ✦ Discuss UPC bar codes, PIN numbers and security codes with respect to CHECK digits where division (sometimes referred to as a multiple of) is sometimes used to check the accuracy of data entry.
- ✦ Discuss how some people use arithmetic relationships and properties to help remember them. For example, some people might create a PIN number for which all of the digits are divisible by 3.

Activity 2: Addresses achievement indicators 1 and 2.

- Ask students to list three numbers divisible by 2. Ask them to describe a test for divisibility by 2. Discuss their divisibility tests. Record a student-agreed upon description of the test as the first entry in a chart of divisibility tests.
- Provide a list of five numbers between 100 and 1000, where some are divisible by 2 and some are not (e.g. 148, 310, 126, 999, 237). Ask students to sort the numbers into two categories, divisible by 2 and not divisible by 2, using a Carroll or Venn diagram (see examples below).

Venn diagram



Carroll diagram

Divisible by 2	Not divisible by 2
148 310 126	999 237

Activity 3: Addresses achievement indicators 1 and 2.

- ✦ Ask students to list three numbers divisible by 5. Ask them to describe a test for divisibility by 5. Discuss their divisibility tests. Record a student-agreed upon description of the test as the second entry in a chart of divisibility tests.
- ✦ Provide a list of five numbers between 100 and 1000, where some are divisible by 5 and some are not (e.g. 148, 310, 126, 999, 405). Ask students to sort the numbers into two categories, divisible by 5 and not divisible by 5, using a Carroll or Venn diagram.

Activity 4: Addresses achievement indicators 1 and 2.

- Ask students to list three numbers divisible by 10. Ask them to describe a test for divisibility by 10. Discuss their divisibility tests. Record a student-agreed upon description of the test as the third entry in a chart of divisibility tests.
- Provide a list of five numbers between 100 and 1000, where some are divisible by 10 and some are not (e.g. 148, 310, 100, 995, 230). Ask students to sort the numbers into two categories, divisible by 10 and not divisible by 10, using a Venn or Carroll diagram.

Activity 5: Addresses achievement indicators 1, 2, and 3.

- Ask students to list three numbers that are divisible by 2, 5, AND 10 (e.g. 10, 20, 30). Discuss why these numbers are divisible by 2, 5, AND 10. Discuss that 2, 5, and 10 are factors of the numbers.
- Ask students to list a number that is divisible by 2 and 5 but NOT by 10. After students give up trying to find the number, discuss why it is impossible to find such a number. [10 is 2 x 5; therefore, if a number is divisible by 2 and 5 it automatically becomes divisible by 10.] Discuss that 2 and 5 are factors of 10.

Activity 6: Addresses achievement indicators 1 and 2.

- Ask students to list three numbers between 100 and 200 that are divisible by 3. Allow them to use a calculator to find the numbers. Ask them to describe a test for divisibility by 3. Assist as needed by providing clues (e.g. try adding digits). Discuss their tests. Record a student-agreed upon description of the test as the fourth entry in a chart of divisibility tests.
- Provide a list of five numbers between 100 and 1000, where some are divisible by 3 and some are not (e.g. 147, 360, 102, 995, 230). Ask students to identify the numbers divisible by 3 and not divisible by 3, using the test for divisibility by 3.

Activity 7: Addresses achievement indicators 1 and 2, and practice.

- Students play the game '*Divisible Numbers*'. The rules are:
  - Form teams of 2 players. Two teams play against each other.
  - Each team takes a turn at rolling three dice. The three numbers showing on the dice are used to build 3-digit numbers.
  - The team throwing the dice tries to make up a number divisible, in turn, by 2, by 5, by 10, by 3 (may not be possible because of the numbers showing on the dice). The team gets one point for each divisible number made. When the team cannot make any more (or any) divisible numbers, the turn passes to the other team.
  - The team that gets 15 points first wins the game.

Activity 8: Addresses achievement indicators 1, 2, and 3.

- Ask students to list three number between 100 and 200 that are divisible by 6. Allow them to use a calculator to find the numbers. Ask them to describe a test for divisibility by 6. Assist as needed by providing clues (e.g. try thinking about prime factors of 6). Discuss their tests. Record a student-agreed upon description of the test as the fifth entry in a chart of divisibility tests.
- Provide a list of five numbers between 100 and 1000, where some are divisible by 6 and some are not (e.g. 126, 360, 102, 857, 234). Ask students to identify the numbers divisible by 6 and not divisible by 6, using the test for divisibility by 6.

Activity 9: Addresses achievement indicators 1, 2, and 3.

- Ask students to list three number between 100 and 200 that are divisible by 9. Allow them to use a calculator to find the numbers. Ask them to describe a test for divisibility by 9. Assist as needed by providing clues (e.g. Is 9 equal to 3 x 3?). Discuss their tests. Record a student-agreed upon description of the test as the sixth entry in a chart of divisibility tests.
- Provide a list of five numbers between 100 and 1000, where some are divisible by 9 and some are not (e.g. 126, 360, 102, 857, 531). Ask students to identify the numbers divisible by 9 and not divisible by 9, using the test for divisibility by 9.
- Ask students to create two 3-digit numbers that are both divisible by 9, using the divisibility test for 9 to help create the numbers. Ask students to add the two numbers and check if the sum is also divisible by 9. Discuss the result.
- Organize students into groups. Ask each group to create a 9-digit number using the digits 1, 2, 3, . . . 9 once each in any order. Ask them to check if the resulting 9-digit number is divisible by 9. Ask each group to present their number and the result of the divisibility test. Discuss the matter. [NOTE: This will seem magical. Any 9-digit number formed by using the digits from 1 to 9 once only is divisible by 9 because the sum of the digits from 1 to 9 is 45 (and that is divisible by 9).]

Activity 10: Addresses achievement indicators 1 and 2.

- Ask students to list three number between 100 and 200 that are divisible by 4. Allow them to use a calculator to find the numbers. Ask them to describe a test for divisibility by 4. Assist as needed by providing clues (e.g. try thinking only about the last two digits). Discuss their tests. Record a student-agreed upon description of the test as the seventh entry in a chart of divisibility tests.
- Provide a list of five numbers between 100 and 1000, where some are divisible by 4 and some are not (e.g. 128, 360, 102, 857, 234). Ask students to identify the numbers divisible by 4 and not divisible by 4, using the test for divisibility by 4.

Activity 11: Addresses achievement indicators 1 and 2.

- Ask students to list three number between 1 000 and 2 000 that are divisible by 8. Allow them to use a calculator to find the numbers. Ask them to describe a test for divisibility by 4. Assist as needed by providing clues (e.g. try thinking only about the last three digits). Discuss their tests. Record a student-agreed upon description of the test as the eighth entry in a chart of divisibility tests.
- Provide a list of five numbers between 1 000 and 2 000, where some are divisible by 8 and some are not (e.g. 6044, 1824, 5432, 8575, 2347). Ask students to identify the numbers divisible by 8 and not divisible by 8, using the test for divisibility by 8.

Activity 12: Addresses achievement indicators 1, 2, and 3, and practice.

- Present three 3-digit numbers. Ask students to find the factors of each number using the divisibility tests.
- Students play the game '*Divisible Numbers*'. The rules are:
  - Form teams of 2 players. Two teams play against each other.
  - Each team takes a turn at rolling four dice. The four numbers showing on the dice are used to build 4-digit numbers.
  - The team throwing the dice tries to make up a number divisible, in turn, by 2, by 3, by 4, by 5, by 6, by 8, by 9, by 10 (may not be possible because of the numbers showing on the dice). The team gets one point for each divisible number made. When the team cannot make any more (or any) divisible numbers, the turn passes to the other team.
  - The team that gets 20 points first wins the game.

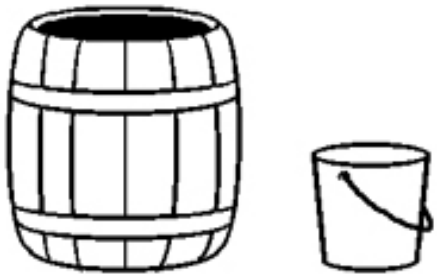
Activity 13: Revisits SET SCENE and addresses achievement indicators 1 and 2, & practice.

- Revisit the SET SCENE task about researching codes. Organize students into pairs.
- Ask each pair to create a 5-digit PIN number that involves at least two divisibility checks to help remember the number.
- Ask selected pairs to present their PIN numbers. Discuss.

Activity 14: Addresses achievement indicator 4.

- ✦ Ask students about dividing by 0. Ask them to predict the answer to 12 ÷ 0 and to 5 ÷ 0. Have them confirm their predictions using a calculator. [Careful about button punching. Enter 12, then ÷, and lastly 0.] [Some calculators will show 'ERROR' on the display; others will display 'error' in other ways.] Discuss the unexpected result (most students will have predicted '0' as the answer to, for example, 12 ÷ 0).
- ✦ Lead students through the following discussion and questions.

1. Ask what division means. STUDENT RESPONSE: splitting up into equal groups.
2. Ask them to imagine 10 cookies and dividing them up into groups of 2. Ask them to describe how the division would work. STUDENT RESPONSE: Make a group of 2, remove it, and keep doing this until all the cookies are gone. The answer is 'the number of groups of 2 removed/formed.
3. Ask them to consider 5 ÷ 0 and to imagine a large barrel of water and a bucket for emptying the barrel. Ask them to explain what the question, "*How many groups are formed?*", means for 5 ÷ 0, and the barrel and pail situation.



4. ENSURE they realize it means that the barrel has 5 litres of water in it and you are using a bucket that holds 0 litres of water to empty it. The question then is: "*How many times do you need to use the bucket to empty the barrel?"*
5. Ask students if this is a futile task. Ensure they realize that you will never be able to empty the barrel because the bucket holds 0 litres of water. You will be trying to empty the barrel forever. This is why the calculator shows the answer to 5 ÷ 0 as ERROR. [The answer to 5 ÷ 0 is sometimes referred to as 'infinity'.]

Activity 15: Assessment of teaching.

- Provide students with a list of six numbers between 200 and 900. Ask students to identify all numbers in the list that are divisible by 2, 3, 4, 5, 6, 8, 9, and 10, using the respective divisibility tests (that are displayed somewhere visible for them to refer to).

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

An example of a partially well-designed worksheet follows.

The worksheet contains a sampling of question types. More questions of each type are needed.

The MAINTAIN stage follows the sample worksheets.

Question 1.

In the list: 23, 85, 902, 610, 47, 236, 522, 555, 764, 123.

Which numbers are divisible by 2? _____

Which numbers are divisible by 5? _____

Which numbers are divisible by 10? _____

Question 2.

In the list: 49, 72, 840, 6 104, 4 775, 2 328, 5 022, 7771, 804, 171.

Which numbers are divisible by 3? _____

Which numbers are divisible by 4? _____

Which numbers are divisible by 8? _____

Question 3.

In the list: 492, 702, 8 406, 6 105, 4 775, 1 101, 666, 7011, 525, 8 803.

Which numbers are divisible by 3? _____

Which numbers are divisible by 6? _____

Which numbers are divisible by 9? _____

Question 4.

In the list: 111, 222, 3 033, 3 144, 6 312, 1 101, 6 663, 7012, 5 256, 8 8034.

Which numbers are divisible by 2? _____

Which numbers are divisible by 3? _____

Which numbers are divisible by 4? _____

Which numbers are divisible by 5? _____

Which numbers are divisible by 6? _____

Which numbers are divisible by 8? _____

Which numbers are divisible by 9? _____

Which numbers are divisible by 10? _____

Which numbers are divisible by 0? _____

Question 5.

List ALL the factors of 260 _____

MAINTAIN stage

Mini-task example

Every so often:

- Present a 4-digit number and ask students to indicate if it is divisible by 2, by 3, by 4, by 5, by 6, by 8, by 9, by 10, by 0.

Rich-task example

Ask students to solve the following problem.

> *Joe wants to make a 6-digit PIN number, using the digits, 1, 2, 3, 4, 5, and 6, once each. He wants a way to help remember the number. He decides to make a special 6-digit number so that the first digit is divisible by 1, the first two digits are divisible by 2, the first three digits are divisible by 3, the first four digits are divisible by 4, the first five digits are divisible by 5, and the entire 6-digit PIN number is divisible by 6. Help Joe find a 6-digit PIN number that meets the divisibility conditions.*

Note:

- The digits referred to in the problem are from the point of view of the left side of the number (100 000s place). For example, the first digit refers to the digit that is the leftmost digit. The first two digits refers to the first two digits on the left side of the number, and so on.
- One solution is: 321 654.

Comments

This is a rich-task because it is a complex problem that integrates divisibility with a real world situation.